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1. Find the center and the radius of
convergence of:

$$\sum_{n=0}^{\infty} \frac{(4n)!}{2^n (n!)^4} (z + \pi i)^n$$

~~Find the center of convergence~~

The center of the series is ~~the~~ the series is

$$z + \pi i = 0, \text{ which is:}$$

Center: $-\pi i$

The series could be expressed as:

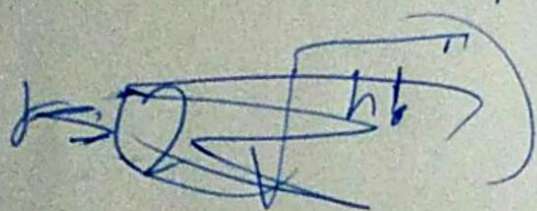
$$\sum_{n=0}^{\infty} C_n (z + \pi i)^n$$

Kennedy's Villainy
 Monday to the Canchy - A & d & d ②
 Theorem the radius of convergence is!

$$r = \frac{1}{\limsup_{n \rightarrow \infty} \sqrt[n]{|c_n|}}$$

applying it to our series!

$$r = \lim_{n \rightarrow \infty} \frac{|c_n|}{|c_{n+1}|}$$



$$r = \lim_{n \rightarrow \infty} \frac{(n!)^{\frac{4}{n}}}{(4n)!^{\frac{1}{5}}}$$

$$r = \lim_{n \rightarrow \infty} \frac{e^{\frac{4}{n} \ln n!}}{e^{\frac{1}{n} \ln (4n)!}} = \lim_{n \rightarrow \infty} \left(e^{\frac{4}{n} \ln n! - \frac{1}{n} \ln (4n)!} \right)$$

using Stirling's approximation

$$(n!) = n \ln n - n + O(\ln n)$$

of limits with $n \rightarrow \infty$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{\frac{4}{n} (n \ln n - n + O(\ln(n)))} - \frac{1}{n} (4n \ln 4n - 4n + O(\ln(4n))) \right)$$

$\lim_{n \rightarrow \infty} \frac{O(\ln(n))}{n} = 0$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{4 \ln n - 4} - (4 \ln 4n - 4) \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{4 \ln n - 4} - 4 \ln 4n - 4 \right)$$

$$r = \lim_{n \rightarrow \infty} 2 \left(e^{-4 \ln 4} \right) = 2 \left(4^{-4} \right) = 2/2^8$$

There fore!

radius of convergence: 2^{-7}

Find the first four terms of the Taylor series!

$e^z (z-2)$ centered at $z_0 = 1$

$f(z) = e^z (z-2)$

$f(1) = e$

$f'(1) = 0$ $f'(z) = 2ze^{z^2-2z} - 2e^{z^2-2z}$

$f''(1) = \frac{2}{e}$ $f''(z) = -8ze^{z^2-2z} + 4z^2e^{z^2-2z} + 6e^{z^2-2z}$

$f'''(1) = 0$ $f'''(z) = (8z^3 - 24z^2 + 36z - 20)e^{z^2-2z}$

$f^{(4)}(1) = \frac{12}{e}$

$f^{(6)}(1) = \frac{120}{e}$

$e^z (z-2) = e + \frac{2}{e} (x-1)^2 + \frac{12}{e} (x-1)^4 + \frac{120}{e} (x-1)^6 + \dots$

$$e + \frac{2}{e} (x-1)^2 + \frac{12}{e} (x-1)^4 + \frac{120}{e} (x-1)^6$$

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2. Find the Power series for $\frac{z+2}{1-z^2}$ following
the hints series!

$$\frac{z+2}{1-z^2}$$

$$\frac{z+2}{1-z^2} = \frac{z+1}{(1-z)(1+z)} + \frac{1}{1-z^2}$$

$$= \frac{1}{1-z} + \frac{1}{1-z^2}$$

from the ^{geometric} series formula!

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\frac{z+2}{1-z^2} = \frac{1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + \dots}{1 + z^2 + z^4 + z^6 + \dots}$$

$$\frac{z+2}{1-z^2} = \boxed{2 + z + 2z^2 + z^3 + \dots}$$

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4. Find the Fourier series of the given function, which is assumed to have the period 2π

$$f(x) = \begin{cases} x & \text{if } (-\pi < x < 0) \\ \pi - x & \text{if } (0 < x < \pi) \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \left(\int_{-\pi}^0 x dx + \int_0^{\pi} (\pi - x) dx \right) \\ &= \frac{1}{2\pi} \left(\frac{x^2}{2} \Big|_{-\pi}^0 + \left(\pi x - \frac{x^2}{2} \right) \Big|_0^{\pi} \right) = -\frac{\pi^2}{2} + \pi^2 - \frac{\pi^2}{2} \\ &= 0 \quad \therefore a_0 = 0 \end{aligned}$$

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$$G_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{P(x)}{f(x)} \cos n\alpha \, d\alpha$$

$$T_3 = \frac{1}{\pi} \left(\int_{-\pi}^0 x \cos n\alpha \, d\alpha + \int_0^{\pi} (\pi - x) \cos n\alpha \, d\alpha \right)$$

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$$x \cos n\alpha$$

$$1 \quad \frac{1}{n} \sin n\alpha$$

$$0 - \frac{1}{n^2} \cos n\alpha$$

$$G_2 = \frac{1}{\pi} \left(\frac{1}{n} \sin n\alpha - \frac{1}{n^2} \cos n\alpha \right) \Big|_{-\pi}^0 + \frac{\pi}{n} \sin n\alpha$$

$$G_{2k} = \frac{1}{\pi} \left(\frac{1}{n^2} - \frac{1}{n^2} \right) + \left(\frac{1}{n^2} - \frac{1}{n^2} \right) = 0$$

$$G_{2k+1} = \frac{1}{\pi} \left(-\frac{1}{n^2} - \frac{1}{n^2} \right) + \left(\frac{1}{n^2} - \frac{1}{n^2} \right)$$

$$= -\frac{4}{n^2 \pi} = \frac{-4}{(2k+1)^2 \pi}$$

$$\frac{1}{n} \sin n\alpha + \frac{1}{n^2} \cos n\alpha$$

Partial Fourier series known

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left(\int_{-\pi}^0 x \sin nx \, dx + \int_0^{\pi} (\pi - x) \sin nx \, dx \right)$$

0 1

$$\sim \sin nx$$

$$1 - \frac{1}{n} \cos nx$$

$$0 - \frac{1}{n^2} \sin nx$$

$$b_n = \frac{1}{\pi} \left(-\frac{1}{n} \cos nx - \frac{1}{n^2} \sin nx \Big|_{-\pi}^0 + \left(-\frac{\pi}{n} \cos nx + \frac{1}{n} \cos nx - \frac{1}{n^2} \sin nx \right) \Big|_0^{\pi} \right)$$

$$b_{2k} = \frac{1}{\pi} \left(-\frac{1}{n} + \frac{1}{n} \right) + \left(\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) = 0$$

$$b_{2k+1} = \frac{1}{\pi} \left(-\frac{1}{n} - \frac{1}{n} \right) + \left(-\frac{1-\pi}{n} - \frac{1-\pi}{n} \right) = 0$$

$$b_{2k+1} = \frac{-4+2\pi}{(2k+1)\pi}$$

$$= \frac{-4+2\pi}{h\pi}$$

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$$f(x) = \sum_{k=1}^{\infty} \left[\frac{-4 \cos((2k-1)x)}{(2k-1)^2 \pi} + \frac{(2\pi-4) \sin((2k-1)x)}{(2k-1) \pi} \right]$$

Therefore, the Fourier series for the function is!

$$\sum_{k=1}^{\infty} \left[\frac{-4 \cos((2k-1)x)}{(2k-1)^2 \pi} + \frac{(2\pi-4) \sin((2k-1)x)}{(2k-1) \pi} \right]$$